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A Digital Reverberator Controlled through Measures of the Reverberation

Zafar RAFII, Bryan PARDO

Abstract

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Five measures commonly used to characterize reverberation are defined from the impulse response, and formulae are derived to estimate values of these reverberation measures in terms of the digital parameters for the reverberator. Dependence relations between the digital parameters lead to the need of only five independent parameters to control the whole reverberator, so that the measures can be redefined as five functions of five independent parameters. This mapping parameters-measures is used to let the reverberator to be controlled through the measures of the reverberation, in other words the reverberator can generate a reverberation effect based on desired characteristics of the reverberation itself.

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Keywords: Reverberation, digital reverberator, comb filter, all-pass filter, Schroeder, Moorer, reverberation time

A DIGITAL REVERBERATOR CONTROLLED THROUGH MEASURES OF THE REVERBERATION

Zafar Rafii

Northwestern University
EECS Department
Evanston, IL 60201, USA
zafarrafii2011@u.northwestern.edu

Bryan Pardo

Northwestern University
EECS Department
Evanston, IL 60201, USA
pardo@northwestern.edu

ABSTRACT

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***Index Terms*—** reverberation, digital reverberator, comb filter, all-pass filter, Schroeder, Moorer, reverberation time

1. INTRODUCTION

1.1. The Reverberation Effect

Reverberation is defined as the persistence of sound in a particular space after the original sound is removed [1]. A reverberation is created by the reflections of a sound in an enclosed space causing a large number of echoes to build up and then slowly decay as the sound is absorbed by the walls and air [2]. It is a common natural phenomenon most noticeable in large spaces, such as concert halls and churches.

The reflections caused by the reverberation modify the perception of the sound, changing its loudness, timbre and spatial characteristics [3], [4]. The issue of how reverberation affects the timbral perception of the sound has been rarely addressed [4], [5]. Most of the studies deal with reverberation in the context and for the purpose of auralization or room simulation [4], [6]. Furthermore, the authors seem often to prefer the use of convolution reverberation for modelling the acoustics of a room [4], [7], [8].

Based on limits of perception, the impulse response of a reverberation can be divided into two segments: the *early reflections* and the *late reverberation*. The early reflections are the relatively sparse first echoes that are directly related to the shape and size of the space, as well as the position of the source and the listener in the space, and thus have a key role in the subjective spatial impression of the room. They arrive after the direct sound and last generally about 50 to 80 msec [8]. The late reverberation is the remainder of reverberation decay, the collection of many reflected sounds which blend and overlap. It is more random and difficult to relate to the physical characteristics of the space, instead it gives statistical impression of room, independent of the source and receiver positions. Fig. 1 represents a typical impulse response of a reverberation.

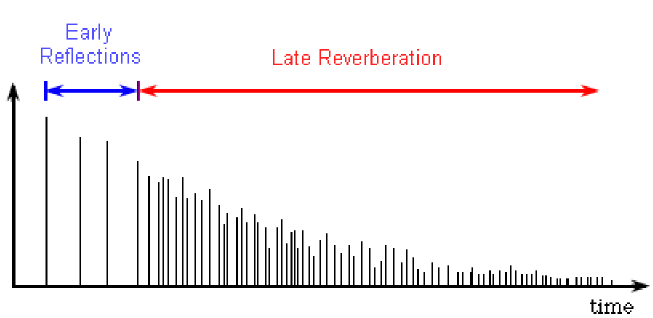

Fig. 1. Typical impulse response of a reverberation.

1.2. Simulating Reverberation

Reverberation tools are typically used to simulate the effect of a sound being placed in a space and are commonly used on commercial audio recordings. A number of systems have been developed to simulate reverberation. The first systems used physical equipments to produce the reverberation effect. The *chamber reverberator* uses a real physical space as a natural echo chamber, where a loudspeaker would play a sound for a microphone to pick it up again, including the reverberation effect of the space. The *plate reverberator* system uses an electromechanical transducer to create vibration in a large plate of sheet metal, then a pickup captures the vibrations on the plate and the result is outputted as an audio signal. The *spring reverberator* uses a metal spring with a transducer at one end and a pickup at the other to create and capture vibrations within the spring.

The advances of signal processing techniques enabled the appearance of *digital reverberators*, easier to manipulate and offering enhanced possibilities in simulating reverberation. Since reverberation is essentially caused by a very large number of echoes, simple digital signal processors can use multiple feedback delay circuits to create a large, decaying series of echoes. More advanced digital reverberators can now simulate the time and frequency domain responses of real rooms based upon the room dimensions, air and walls absorption and other properties. Typical digital audio editors incorporate one or several digital tools to create reverberation. For example, Fig. 2 shows the interface of the *Platinumverb* reverberation tool available in the popular *Logic Audio* production suite.

Fig. 2. *Logic Audio*'s *Platinumverb* interface.

In audio signal processing, another method based on the mathematical convolution operation is also used for digitally simulating the reverberation of a physical or virtual space. The *convolution reverberation* uses the impulse response of the space being modelled to convolve it with the incoming audio signal to be processed to simulate the reverberation effect of that space [4], [7], [8]. Some softwares such as *Altiverb* proposes to use convolution reverberation to simulate the reverberation of particular rooms or concert halls.

1.3. The approach

In this study, we have decided to develop a digital reverberator based on digital filters to simulate reverberation. Digital reverberators are easier to manipulate through their digital parameters, they need less computation and storage than convolution reverberation methods, and give satisfactory sound quality [9].

Inspired from previous classic works done on reverberation simulation, we have decided to build a simple but efficient stereo reverberation control, composed of simple digital filters typically used in simulating reverberation, such as the comb filter and the all-pass filter. The reverberator incorporates also a low-pass filter to simulate the air and walls absorption effect, a gain parameter to control the wet/dry effect, and a delay difference parameter to introduce a difference between the channels. Dependence relations between all the digital parameters let the reverberator to be eventually controlled using only five independent parameters.

Yet, we would like to manipulate the reverberator and generate reverberation using meaningful and not too abstract controls, such as characteristics of the desired reverberation effect. Therefore, five measures of the reverberation, defined from the impulse response and commonly used to characterize reverberation, are derived and mapped to the parameters of the digital reverberator, such that the reverberator can be finally controlled through those reverberation measures.

The paper is organized as follows. Section 2 introduces three digital filters commonly used for simulating reverberation and section 3 presents three classic models of digital reverberators. Based on those digital filters and inspired from those previous works, a reverberator is described and developed in section 4. Five reverberation measures are defined and mapped to the parameters of the reverberator in section 5. Finally, conclusion and perspectives are given in section 6.

2. DIGITAL FILTERS FOR REVERBERATION

In this section, three digital filters commonly used for simulating reverberation are introduced and analyzed, in time through their impulse response and in frequency through their transfer function.

2.1. The Comb Filter

Looking at the impulse response of Fig. 1, an immediate first simple digital model of reverberation would be an infinite series of delta functions exponentially decaying. A digital filter with an impulse response of that kind exists and is called *comb* filter [10].

Fig. 3 and 4 show respectively the scheme of the comb filter and its impulse response. The comb filter is a simple delay d with a feedback of gain g. As we can notice on its impulse response, the delay factor d determines the echo spacing.

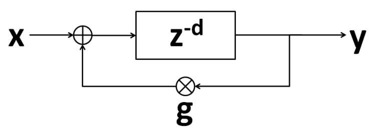

Fig. 3. Scheme of the comb filter.

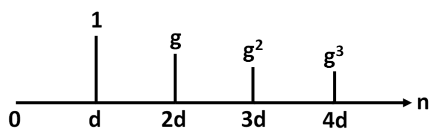

Fig. 4. Impulse response of the comb filter.

Eq. 1 and 2 represent respectively the output and the transfer function of the comb filter.

$$y[n] = x[n-d] + g\,y[n-d] \tag{1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-d}}{1 - g\,z^{-d}} \tag{2}$$

Looking at the impulse response of Fig. 4, a discrete temporal function for the comb filter can be deduced. Eq. 3 represents the amplitude of the comb filter in function of the discrete time.

$$A[n] = g^{\frac{n}{d}-1} \text{ for } n = d,\ 2d, ... \tag{3}$$

The comb filter is however too simplistic to model realistic reverberation. First, the impulse response in Fig. 1 has equally spaced peaks, whereas natural reverberation has increasingly dense peaks with time. Furthermore, its frequency response periodically drops to a local minimum (notch), and periodically rises to a local maximum (peak), resulting in a comb-like shape (hence the name). This introduces a coloration which results in a kind of "ringing" noise in the filter.

2.2. The All-pass Filter

An alternative to solve this last problem would be to use an *all-pass* filter [10]. As its name indicates, the all-pass filter passes all frequencies, resulting in a flat magnitude frequency response. However, it does affect the phase of the signal which is not flat, but this is not really a problem since the human auditory system is not very sensitive to phase.

Fig. 5 and 6 show respectively the scheme of the all-pass filter and its impulse response. As we can notice, the all-pass filter is like a comb filter with a feedforward of gain $-g$ around the delay. And like the comb filter, the delay d determines the echo spacing. Note that the first peak of the impulse response of the all-pass filter is negative. In order the first peak to fit in with the others in terms of the exponential decay, we need $1 - g^2 = g^2$ that is to say $g = \frac{1}{\sqrt{2}}$.

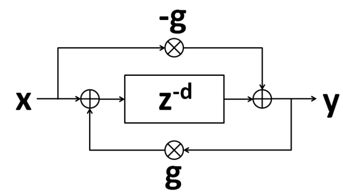

Fig. 5. Scheme of the all-pass filter.

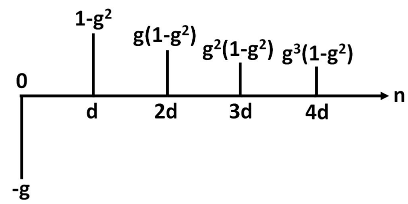

Fig. 6. Impulse response of the all-pass filter.

Eq. 4 and 5 represent respectively the output and the transfer function of the all-pass filter.

$$y[n] = x[n-d] - g\,x[n] + g\,y[n-d] \tag{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-g + z^{-d}}{1 - g\,z^{-d}} \tag{5}$$

Looking at the impulse response of Fig. 6, a discrete temporal function for the all-pass filter can be deduced. Eq. 6 represents the amplitude of the all-pass filter in function of the discrete time.

$$A[n] = \begin{cases} -g & \text{for } n = 0 \\ (1-g^2)g^{\frac{n}{d}-1} & \text{for } n = d,\ 2d, ... \end{cases} \tag{6}$$

Listening to the results, we can notice that this filter has a more pronounced reverberation effect. This may be due to the first negative peak, so the direct sound will be out of phase. However, the all-pass filter has still equally spaced peaks, and it is still a too simple model of reverberation.

2.3. The Low-pass Filter

Besides of being too simplistic models, the filters described above do not take into account the tendency of the reverberation to attenuate higher frequencies, corresponding to the air and walls absorption effect. To solve this problem, *low-pass* filter is commonly incorporated to the other filters.

Fig. 7 and 8 show respectively the scheme of the low-pass filter and its impulse response. A simple first-order low-pass filter with a gain g is sufficient to get satisfactory results.

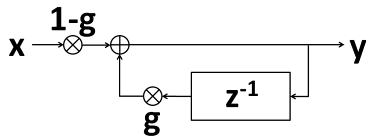

Fig. 7. Scheme of the low-pass filter.

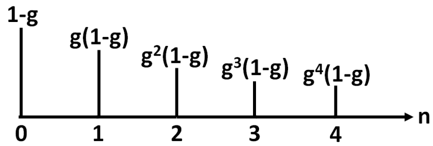

Fig. 8. Impulse response of the low-pass filter.

Eq. 7 and 8 represent respectively the output and the transfer function of the low-pass filter.

$$y[n] = (1 - g)\, x[n] + g\, y[n - 1] \quad (7)$$

$$H(z) = \frac{1 - g}{1 - g\, z^{-1}} \quad (8)$$

Looking at the impulse response of Fig. 8, a discrete temporal function for the low-pass filter can be deduced. Eq. 9 represents the amplitude of the low-pass filter in function of the discrete time.

$$A[n] = (1 - g)\, g^n \;\text{ for } n = 0,\ 1,\ 2, ... \quad (9)$$

The incorporation of low-pass filter helps to simulate a more "natural" reverberation, avoiding "unnatural metallic" sounding. However, even if the results are pretty good, using a single digital filter, even enhanced with a low-pass filter, is not enough to get a "realistic" reverberation.

3. CLASSIC DIGITAL REVERBERATORS

In this section, three classic reverberators based on the digital filters described in section 2 are presented and analyzed. Those digital reverberators are the first simple but efficient models of reverberation that have been proposed, and most of the work done on digital reverberation simulation is based on those models.

3.1. Schroeder's Reverberators

By combining several of the digital filters described in section 2, it is possible to achieve a more "natural" reverberation. Schroeder first proposed to use five all-pass filters in series with delay times that are incommensurate [10]. Each all-pass expands each impulse from the previous stage into an entire infinite all-pass impulse response. And as a series of all-pass filters is an all-pass filter, the whole unit still produces a "colorless" reverberation. Respectively, delay times of 100, 68, 60, 19.7 and 5.85 msec and gain values of 0.7, −0.7, 0.7, 0.7 and 0.7 were used.

Later, Schroeder arranged his design by using four comb filters in parallel, followed by two all-pass filters in series. This design, perhaps the most popular, is known as *Schroeder Reverberator* [11]. The parallel comb filters are supposed to simulate the complex modal response of a room by adding echoes together and reduce the spectral coloration. The range of delay times of the comb filters is between 30 and 50 msec and relatively prime to one another in order to avoid at maximum overlapping echoes. Typical values are 29.7, 37.1, 41.1 and 43.7 msec. Note that the delay and the gain factors of a filter combine to determine the *reverberation time* (see section 5.1) of that filter. In Schroeder Reverberator, the reverberation times of the comb filters, which control the reverberation time of the entire unit, should be set equal. The corresponding gain values can then be easily deduced.

As for the all-pass filters, they allow to increase the *echo density* (see section 5.2) produced by the comb filters in order to give a more "natural sounding" reverberation. Typical values are respectively 96.83 and 32.92 msec for the delay times, and 5 and 1.7 msec for the reverberation times. Fig. 9 represents the scheme of Schroeder Reverberator.

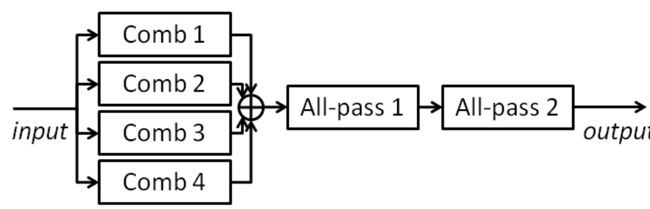

Fig. 9. Schroeder's second reverberator, commonly known as Schroeder Reverberator.

3.2. Chamberlin's Reverberator

Similarly to Shroeder's first reverberator, Chamberlin used five all-pass filters in series, with the last two filters doubled in parallel to get a stereo output in order to simulate a stereo reverberation [12]. He also used un gain parameter to control the overall gain of the reverberation, and finally summed the outputs of the stereo reverberation process to the stereo direct sound.

The delays parameters should be approximately exponentially distributed but in all cases must be a prime number of samples. As a first approximation, Chamberlin proposed to give the first stage the longest delay, which is in the 50-msec range, and then successively multiply it by a constant somewhat less than 1. He precised that the last delay, the shortest one, should not be much less than 10 msec if a distant, hollow sound is to be avoided. As for the gain values, they tend to be similar for all stages, although they should not be identical. Chamberlin explained that, for practical purposes, the reverberation time of a cascade of sections is equal to the longest section time.

Finally, to simulate a stereo reverberation, a subtle difference was introduced between the parameters of each channel to insure that the reverberation will be perceived as coming from all directions, while the original signal retains its normal directivity. Fig. 10 represents the scheme of Chamberlin's reverberator.

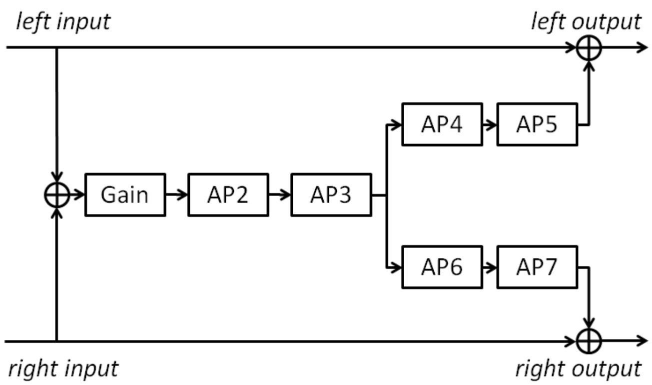

Fig. 10. Chamberlin's reverberator.

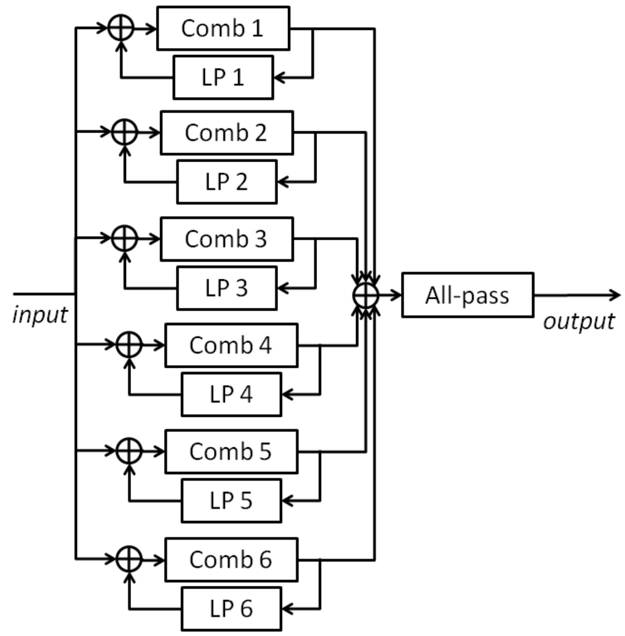

Fig. 11. Moorer's reverberator.

3.3. Moorer's Reverberator

Moorer have studied Schroeder's previous work and came with a new digital reverberator [13]. He proposed to use six comb filters in parallel followed by a single all-pass filter. To simulate the attenuation of higher frequencies by the air, he incorporated a first-order low-pass filter in the loop of each comb filter.

The delay values of the comb filter seem to work well when distributed linearly over a ratio of 1:1.5, with a recommended range of 50 to 80 msec. Moorer precised that the shortest delay can be reduced to 10 msec without gross degradation. The delay lengths in samples should be set to the closest prime numbers to prevent exactly overlapping echoes. The gain values of the comb filters are obtained from the gain values of the low-pass filters and the corresponding reverberation times which are all set to be equal to the overall reverberation time.

As for the all-pass filter, it seems to be sensitive to background noise with too short delay values and seems to produce an audible repetition period with any delay longer than 6 msec. As a consequence, the delay time of the all-pass filter is pretty well limited to 6 msec. As for the gain value, 0.7 seems to work well. Fig. 11 represents the scheme of Moorer's reverberator.

Following an idea reported by Schroeder, Moorer also considered to directly simulate the N early echoes by a N-tap finite impulse response filter section, and simulate the late response with a standard digital reverberator such as the one he proposed, in order to artificially reproduce in a more realistic way the geometry of real rooms. With his model, Moorer achieved a good-sounding, smooth artificial reverberation which has eliminated some of the problems related to earlier digital reverberators, which used to exhibit "fluttery" or "metallic" ringing.

Moorer finally noted that recirculating delay reverberators have a characteristic sound affecting the timbre, and referred to a conceivable further study where the physical characteristics of the sound could be correlated with the subjective perception of the sound.

4. THE DEVELOPED REVERBERATOR

In this section, we describe the digital reverberator we have decided to develop, based on the digital filters introduced in section 2 and inspired from the previous works presented in section 3. A first description of the reverberator is given, followed by a specification including dependence relations between the parameters and ranges of values. Finally, possible other improvements are presented, including other enhanced classic digital reverberators.

4.1. Description

We have developed a digital reverberation unit, mostly inspired by Moorer's work [13]. The reverberator is composed of a block of six *comb* filters in parallel used to simulate the complex modal response of a room by adding echoes together. Each comb filter is characterized by two parameters, a delay factor d_k and a gain factor g_k ($k = 1..6$). The reverberator takes as input a stereo sound whose average over its channels is sent to each comb filters.

The outputs of the comb filters are summed and sent to an *all-pass* filter used to increase the *echo density* (see section 5.2) produced by the comb filters and doubled into two channels to simulate a more "natural sounding" reverberation in stereo. Similarly, the all-pass filter at each channel is characterized by a delay factor d_7, respectively d_8, and a gain factor g_7, respectively g_8.

To simulate air and walls absorption effects, a single first-order low-pass filter defined through its cut-off frequency f_c is added after the all-pass filter at each channel. Following the low-pass filter, a gain parameter G allows to control the wet/dry effect of the reverberation at each channel. A small difference m is finally introduced between the delays of the all-pass filters to insure a difference between the channels for the reverberation. At the end, the outputs of the reverberation process are added to the corresponding channels of the stereo direct sound. Fig. 12 shows the whole reverberation unit.

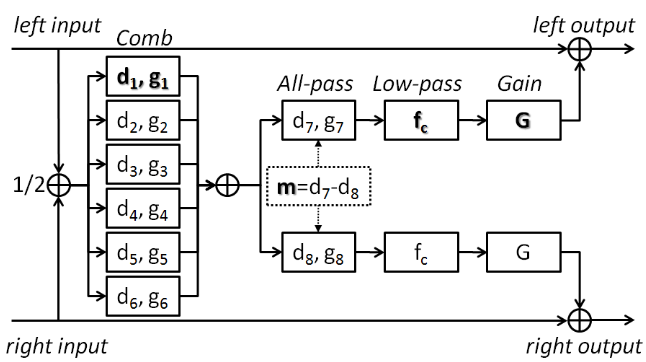

Fig. 12. The digital reverberation unit.

4.2. Specification

There are then six delay parameters and six gain parameters controlling the block of parallel comb filters, in other words 12 control parameters. According to Moorer, the delay values are distributed linearly over a ratio of 1:1.5. The first comb filter is defined as having the longest delay d_1, so the delays of the other comb filters can be deduced from d_1. Based on previous studies (see section 3.3) and our own experiments, the range of values for the delays of the comb filters is defined between 10 and 100 msec. This range is extended to the values of d_1. Furthermore, according to Moorer, the *reverberation times* (see section 5.1) of all the comb filters are set to be equal. Therefore, the gain values for the comb filters, can all be deduced from the reverberation time and the delay value of the first comb filter. In particular, the gain factor of the first comb filter g_1 represents the smallest gain and has a non-inclusive range of values defined between 0 and 1. Although the comb filter gives a non-flat frequency response, a sufficient number of comb filters in parallel with equal values of reverberation time helps to produce an approximated flat frequency response. In the end, only two parameters are needed to control the whole block of parallel comb filters: the delay factor d_1 and the gain factor g_1 of the first comb filter.

Note that the direct sound is assumed to be at time 0, so to avoid the first peak of the reverberation to occur at time 0, the original comb filter of section 2.1 is delayed of d samples (the k^{th} comb filter is delayed of d_k samples). As a consequence, the first peak of the impulse response of the reverberation unit occurs at time d_6, the shortest delay value among the six parallel comb filters. Eq. 10 and 11 represent respectively the transfer function and the discrete temporal function of the new delayed k^{th} comb filter.

$$H_k(z) = \frac{g_k\ z^{-d_k}}{1 - g_k\ z^{-d_k}} \tag{10}$$

$$A_k[n] = g_k^{\frac{n}{d_k}} \ \text{ for } n = d_k,\ 2d_k, ... \tag{11}$$

According to Moorer, the original delay factor d_a of the all-pass filters is fixed to 6 msec. Since a difference of m is added between the delays of the all-pass filters at each channel, the delays become respectively $d_7 = d_a + \frac{m}{2}$ for the left channel and $d_8 = d_a - \frac{m}{2}$ for the right channel, so that $m = d_7 - d_8$. Therefore, the range of values for m is defined between 0 and 12 msec (non-including). Note that, as explained in section 2.2, it is assumed that all-pass filters do not introduce coloration. However, this assumption is valid from a perceptual viewpoint only if the delay line of the all-pass filter is much shorter than the integration time of the ear, which is about 50 msec [14]. For practical purposes, the gain factor of the all-pass filters is fixed to $g_a = \frac{1}{\sqrt{2}}$ for both channels. In other words, $g_7 = g_8 = g_a$. Thus, as explained in section 2.2, the impulse response of the all-pass filter shows a convenient whole exponential decay, which would also help to simplify further computations (see section 5). The discrete temporal function of the all-pass filter in Eq. 6 is then reduced to the following Eq. 12 (in absolute value).

$$|A_a[n]| = g_a^{\frac{n}{d_a}+1} \ \text{ for } n = 0,\ d_a,\ 2d_a, ... \tag{12}$$

Therefore, a single parameter control the all-pass filters at each channel: the difference between the delay values m. Note that to prevent exactly overlapping echoes, the delay values for the comb filters and the all-pass filters are set to the closest inferior prime number of samples.

For practical purposes, the low-pass filter following the all-pass filter at each channel is defined from its cut-off frequency parameter f_c [15]. f_c has a non-inclusive range of values defined between 0 and half of the frequency sampling f_s. Eq. 13 represents the gain g_c of the low-pass filter (see section 2.3) computed from the corresponding cut-off frequency f_c and frequency sampling f_s.

$$g_c = 2 - \cos\left(2\pi\frac{f_c}{f_s}\right) - \sqrt{\left(\cos\left(2\pi\frac{f_c}{f_s}\right) - 2\right)^2 - 1} \tag{13}$$

As for the gain parameter *G* following the low-pass filter at each channel, it is a simple temporal multiplicative constant whose range of values is defined between 0 and 1.

In summary, a total of only five independent digital parameters are needed to control the whole reverberation unit and generate reverberation (The other parameters can be deduced from them according to the relations described above):

1. The longest delay factor, of the first comb filter: $\boldsymbol{d}_1$
2. The smallest gain factor, of the first comb filter: $\boldsymbol{g}_1$
3. The delay difference between the all-pass filters at each channel: $\boldsymbol{m}$
4. The cut-off frequency of the low-pass filters: $\boldsymbol{f}_c$
5. The wet/dry gain parameter: $\boldsymbol{G}$

4.3. Other Improvements

Although the results are convincing reverberation effects, other improvements can still be made to produce an even more realistic reverberation. More filters can be added to achieve a greater echo density, or complex structures can be used to obtain a more "natural" sounding.

For example, Smith proposed the *Digital Waveguide Networks* [16]. The idea is to build a network of bidirectional delay lines simulating wave propagation in a duct capable of producing the desired early reflections and a diffuse, sufficiently dense late reverberation. Another example is the *Feedback Delay Networks* proposed by Jot and Chaigne [9]. They constitute a generalization of comb filters, where the number of feedback paths have been increased using matrices. A last example is the *Late Reverberator* of Gardner which uses the idea of nested all-pass filters [6].

Although, those improvements allow to achieve a more realistic reverberation, they also lead to more complex reverberators with multiple parameters, more complicated to manipulate.

5. THE REVERBERATION MEASURES

In this section, five measures commonly used to characterize reverberation, and defined from the impulse response of the reverberator are presented. Because those measures are more meaningful and less abstract representations of the reverberation, they are mapped to the control parameters of the reverberator, so that the reverberator can eventually be controlled using the measures of the reverberation. Formulae are derived from the definitions of the measures in terms of the parameters of the digital filters, and the reverberation measures are eventually mapped to the five independent parameters defined in section 4.2.

5.1. The Reverberation Time

The *Reverberation Time*, T_{60}, is defined as the time in seconds required for the reflections of a direct sound to decay by 60 dB below the level of the direct sound [17].

Note that the loudest crescendo for most orchestral music is about 100 dB and a typical room background level for a good music-making area is about 40 dB, thus the standard reverberation time can be seen as the time for the loudest crescendo of the orchestra to decay to the level of the room background [17].

The reverberation time for the comb filter and the all-pass filter defined in section 4 can be easily computed from their discrete temporal function, respectively Eq. 11 and Eq. 12, using the definition of the reverberation time above. Eq. 14 and 15 represent respectively the reverberation time for the comb filter and the all-pass filter. Note that, for both comb and all-pass filters, if the delay factor and the reverberation time are given, the gain factor would be easy to deduce.

$$T_{60k} = d_k \, \frac{\log 10^{-3}}{\log g_k} \tag{14}$$

$$T_{60a} = d_a \, \left(\frac{\log 10^{-3}}{\log g_a} - 1 \right) \tag{15}$$

The true value of the reverberation time of the reverberation unit is measured from the impulse response of the reverberation process, so without the direct sound at time $t = 0$. Basically, the function looks for the last peak which has an amplitude above 10^{-3}, which corresponds to -60 dB compared to the direct sound. Note that in discrete time, the direct sound is represented as a Kronecker delta function of amplitude 1. The reverberation time would then be defined as the next sample to that detected peak, in seconds.

To estimate the reverberation time directly from the parameters of the reverberator, the impulse response of the whole reverberation unit needs to be seen as the sum of the impulse responses of six parallel combinations of one comb filter, one all-pass filter, one low-pass filter and one wet/dry gain filter in series, plus the direct sound which does not need to be considered here since it is not included in the computation of the reverberation time. For each combination, every peak of the comb impulse response is expanded into a whole all-pass impulse response. Note that since prime values of delay have been used for the comb filters and the all-pass filters (see section 4), the echoes are assumed not to overlap (or almost not). The effect of the next low-pass filter can be approximated as an additional gain of $(1 - g_c)$ in the impulse response corresponding to the amplitude gain of the low-pass filter (see Eq. 8). The amplitude of the impulse response is further modified by a factor *G* corresponding to the wet/dry gain. By making the approximation that the all-pass filter simply adds an additional gain of g_a in the impulse response corresponding to the gain factor of the all-pass filter, every parallel combination can finally be seen as simple comb filter times an overall amplitude gain value of $g_a\ (1 - g_c)\ G$. Therefore, an approximation of the reverberation time for every parallel combination can be computed from the definition of the reverberation time of the comb filter in Eq. 14.

The reverberation time for the whole reverberation unit is finally estimated as the maximum of the reverberation times of the parallel combinations, as shown in Eq. 16.

$$\boxed{T_{60} = \max_{k=1..6} \left(d_k \, \frac{\log\left(\frac{10^{-3}}{g_a\ (1-g_c)\ G}\right)}{\log g_k} \right)} \tag{16}$$

5.2. The Echo Density

The *Echo Density*, D_t, is the frequency of occurring peaks at a certain time t of the impulse response, in other words it is defined as the number of echoes per second at a time t [18].

For one comb filter or one all-pass filter, the echo density is independent from the time t and represents the delay value of the filter. Indeed sections 2.1 and 2.2 have shown that the delay factor determines the (constant) echo spacing in the impulse response of the filter. For parallel combinations of comb or all-pass filters, the echo densities add, independently of t. In the case of two comb and/or all-pass filters in series, the echoes overlap so that there is approximatively a linear echo density buildup with time. Eq. 17 shows the estimation of the echo density for two filters in series with respective delay factors of d_α and d_β.

$$D_t = \frac{t}{d_\alpha d_\beta} \tag{17}$$

By assuming that the low-pass filter does not introduce any echo and since the wet/dry gain does not affect the echo spacing, the echo density of the whole reverberation unit can be seen as the echo density of six parallel combinations of one comb filter and on all-pass filter in series, not including the peak at $t = 0$ corresponding to the direct sound. Therefore, the echo density of the whole reverberation unit can be estimated as follows, in Eq. 18. Note that the echo density is independent of any gain factor. For practical purposes, the echo density is computed at time $t = 100$ msec and defined as $D = D_t$.

$$\boxed{D = \frac{t}{d_a} \sum_{k=1}^{6} \frac{1}{d_k}} \tag{18}$$

The true value of the echo density of the reverberation unit is measured from the impulse response of the reverberation process, so without the direct sound at time $t = 0$. Basically, the function computes the number of peaks from the time 0 till the time t, and divide this number by t to get the number of peaks per second.

Note that if the echo density is larger than 20-30 echoes per second, the ear no longer hears the echoes as separate events, but fuses them into a sensation of continuous decay [18]. In other words, the early reflections become a late reverberation.

5.3. The Clarity

The *Clarity*, C_t, describes the ratio in dB of the energies in the impulse response p before and after a given time t [7]. The definition of C_t in discrete time is given by the following Eq. 19.

$$C_t = 10 \log_{10} \frac{\sum_{n=0}^{t} p^2[n]}{\sum_{n=t}^{\infty} p^2[n]} \tag{19}$$

As its name suggests, the clarity can provide indication of how “clear” the sound is at time t. This time is usually referred to the arrival of the direct sound or the time when the early reflections give way to the late reverberation. t is usually set to 50 msec for speech (C_{50}) and 80 msec for music (C_{80}) [7].

For practical purposes, the clarity is here computed at time $t = 0$, corresponding to the arrival time of the direct sound. In other words, the clarity represents now the ratio in dB between the energy of the direct sound and the energy of the reverberation. Since the direct sound is here represented as a Kronecker delta function of amplitude 1, the definition of the clarity becomes the minus of the energy of the reverberation in dB, and the Eq. 19 is reduced to the following Eq. 20. Note that the linear clarity (not in dB) is called *Definition* and can be represented here simply as the inverse of the energy of the impulse response.

$$C = C_0 = -10 \log_{10} \sum_{n=0}^{\infty} p^2[n] \tag{20}$$

The true value of the clarity of the reverberation unit is measured from the impulse response of the reverberation process, so without the direct sound at time $t = 0$, by using the Eq. 20 above.

To estimate the clarity directly from the parameters of the reverberator, again the impulse response of the whole reverberation unit needs to be seen as the sum of the impulse responses of six parallel combinations of filters, plus the direct sound which does not need to be considered here since it is not included in the computation of the clarity. Since prime values of delay have been used for the comb filters and the all-pass filters so that the echoes do not overlap (or almost not), it can be assumed that the sum of the squared amplitude of the peaks of the impulse response of the whole reverberation process is equal to the sum of the squared amplitude of the peaks of the impulse responses of the parallel combinations summed over all the six combinations. This assumption is almost not altered by the low-pass filter effect. In other words, the total energy of the impulse response of the six parallel combinations can be considered to be equal to the sum of the energies of the impulse responses of the combinations. Furthermore, it can be shown that the total energy of the impulse response of filters in series is equal to the product of the energies of the impulse responses of the filters. As a consequence, the total energy of the impulse response for the whole reverberation process can be approximated as a linear combination of the energies of the impulse responses of the filters which compose it.

Eq. 21 represents respectively the energies of the impulse responses for one comb filter, the all-pass filter, the low-pass filter and the wet/dry gain filter, all computed from their respective discrete temporal functions from section 4. Note that since the gain factor of the low-pass filter has been fixed to $\frac{1}{\sqrt{2}}$, the energy of the impulse response of the low-pass filter is equal to 1. Note also that the clarity for each of those filters can be easily computed from their respective energies of their impulse responses as follows $c = -10\,\log_{10} e$.

$$\begin{cases} e_k = \sum_{n=0}^{\infty} {A_k}^2[n] = \sum_{i=1}^{\infty} ({g_k}^2)^i \underset{0<g_k<1}{=} \frac{{g_k}^2}{1-{g_k}^2} \\ e_a = \sum_{n=0}^{\infty} {A_a}^2[n] = \sum_{j=0}^{\infty} ({g_a}^2)^{j+1} \underset{0<g_a<1}{=} \frac{{g_a}^2}{1-{g_a}^2} = 1 \\ e_c = \sum_{n=0}^{\infty} {A_c}^2[n] = \sum_{l=0}^{\infty} (1-g_c)^2 ({g_c}^2)^l \underset{0<g_c<1}{=} \frac{1-g_c}{1+g_c} \\ e_G = G^2 \end{cases} \tag{21}$$

Eq. 22 represents the total energy of the impulse response of the whole reverberation process, in other words the six parallel combinations of one comb filter, the all-pass filter, the low-pass filter and the wet/dry gain filter in series.

$$E = \sum_{k=1}^{6} (e_k\ e_a\ e_c\ e_G) = G^2\ \frac{1-g_c}{1+g_c}\ \sum_{k=1}^{6} \frac{{g_k}^2}{1-{g_k}^2} \tag{22}$$

The clarity of the whole reverberation unit is finally estimated as the minus of the sum of the energies of the impulse responses of the six parallel combinations in dB, as shown in Eq. 23. Note that the clarity is independent of any delay factor.

$$\boxed{C = -10\,\log_{10}\left(G^2\ \frac{1-g_c}{1+g_c}\ \sum_{k=1}^{6} \frac{{g_k}^2}{1-{g_k}^2}\right)} \tag{23}$$

5.4. The Central Time

The *Central Time*, T_C, is the "center of gravity" of the energy in the impulse response p, defined as follows in discrete time by Eq. 24 [7].

$$T_C = \frac{\sum_{n=0}^{\infty} n p^2[n]}{\sum_{n=0}^{\infty} p^2[n]} \tag{24}$$

The true value of the central time of the reverberation unit is measured from the impulse response of the reverberation process, so without the direct sound at time $t = 0$, by using the Eq. 24 above.

To estimate the central time directly from the parameters of the reverberator, the impulse response of the whole reverberation unit needs to be seen as the sum of the impulse responses of a series formed of the combination of six comb filters in parallel, the all-pass filter, the low-pass filter and the wet/dry gain filter, plus the direct sound which does not need to be considered here since it is not included in the computation of the central time. Since the echoes do not overlap, the same assumption made for the clarity is used here, namely "the square of the sum is equal to the sum of the squares". Based on this assumption and by using the definitions of the clarity in Eq. 23 and the discrete temporal function of the comb filter in Eq. 11, the central time of the combination of six parallel comb filters can be deduced as shown in Eq. 25. Note that for one single comb filter, the central time would be equal to $t_{C\,k} = \frac{d_k}{1-{g_k}^2}$.

$$t_C = \frac{\sum_{k=1}^{6}\sum_{n=0}^{\infty} n{A_k}^2[n]}{\sum_{k=1}^{6}\sum_{n=0}^{\infty} {A_k}^2[n]} = \frac{\sum_{k=1}^{6}\sum_{i=1}^{\infty} i d_k ({g_k}^2)^i}{\sum_{k=1}^{6}\sum_{i=1}^{\infty} ({g_k}^2)^i} = \frac{\sum_{k=1}^{6} \frac{d_k {g_k}^2}{(1-{g_k}^2)^2}}{\sum_{k=1}^{6} \frac{{g_k}^2}{1-{g_k}^2}} \tag{25}$$

Based on the same assumption, it can be shown that the central time of a series of filters is equal to the sum of the central times of the filters. By using the definitions of the clarity and the discrete temporal functions of the all-pass filter in Eq. 12 and the low-pass filter in Eq. 9, their respective central times can be deduced as shown in Eq. 26. Note that since the gain factor of the low-pass filter has been fixed to $\frac{1}{\sqrt{2}}$, the central time of the low-pass filter is equal to d_a. Note also that the central time is equal to 0 for the wet/dry gain filter.

$$
\begin{cases}
t_{C\,a} = \dfrac{\displaystyle\sum_{n=0}^{\infty} n{A_a}^2[n]}{\displaystyle\sum_{n=0}^{\infty} {A_a}^2[n]} = \dfrac{\displaystyle\sum_{j=0}^{\infty} j d_a ({g_a}^2)^{j+1}}{\displaystyle\sum_{j=0}^{\infty} ({g_a}^2)^{j+1}} = \dfrac{d_a {g_a}^2}{1-{g_a}^2} = d_a \\
t_{C\,c} = \dfrac{\displaystyle\sum_{n=0}^{\infty} n{A_c}^2[n]}{\displaystyle\sum_{n=0}^{\infty} {A_c}^2[n]} = \dfrac{\displaystyle\sum_{l=0}^{\infty} l \frac{1}{f_s} (1-g_c)^2 ({g_c}^2)^l}{\displaystyle\sum_{l=0}^{\infty} (1-g_c)^2 ({g_c}^2)^l} = \dfrac{1}{f_s} \dfrac{1}{1-{g_c}^2}
\end{cases}
\tag{26}
$$

The central time of the whole reverberation unit is finally estimated as the sum of the central times of the combination of six parallel comb filters, the all-pass filter and the low-pass filter, as shown in Eq. 27. Note that the central time is independent of any gain amplitude. Note also that the last term corresponding to the central time of the low-pass filter is very small compared to the other terms and can be neglected.

$$
\boxed{T_C = \frac{\displaystyle\sum_{k=1}^{6} \frac{d_k {g_k}^2}{(1-{g_k}^2)^2}}{\displaystyle\sum_{k=1}^{6} \frac{{g_k}^2}{1-{g_k}^2}} + d_a + \frac{1}{f_s}\frac{1}{1-{g_c}^2}}
\tag{27}
$$

5.5. The Spectral Centroid

The *Spectral Centroid*, S_C, is the center of gravity of the energy in the magnitude spectrum P of the impulse response p, defined as follows in discrete time with the sampling frequency f_s by Eq. 28. Perceptually, it has a robust connection with the impression of "brightness" of a sound [19].

$$
S_C = \frac{\displaystyle\sum_{n=0}^{f_s/2} nP^2[n]}{\displaystyle\sum_{n=0}^{f_s/2} P^2[n]}
\tag{28}
$$

The true value of the spectral centroid of the reverberation unit is measured from the magnitude of the impulse response of the reverberation process, so without the direct sound at time $t = 0$, by using the Eq. 28 above.

Although the comb filter has a non-flat magnitude frequency response, with a sufficient number of comb filters in parallel, with equal values of reverberation time, an approximated flat frequency response can be achieved. Since the all-pass filter has a flat magnitude frequency response, only the low-pass filter is assumed to affect the frequency response of the whole reverberation unit.

To estimate the spectral centroid directly from the parameters of the reverberator, the magnitude frequency response of the low-pass filter is first computed from its transfer function in Eq. 8, as shown in Eq. 29

$$
H_c(n) = |H_c(e^{j2\pi n})| = \frac{1-g_c}{\sqrt{1+{g_c}^2 - 2\,g_c\cos(2\pi n)}}
\tag{29}
$$

The spectral centroid of the whole reverberation unit is finally estimated using the definition of the spectral centroid of Eq. 28 and the magnitude frequency response of Eq. 29 as follows in Eq. 30. Note that g_c is computed from f_c so that the spectral centroid is assumed to only depend on the cut-off frequency parameter.

$$
\boxed{S_C = \frac{\displaystyle\sum_{n=0}^{f_s/2} \frac{n}{1+{g_c}^2 - 2\,g_c\cos(2\pi n)}}{\displaystyle\sum_{n=0}^{f_s/2} \frac{1}{1+{g_c}^2 - 2\,g_c\cos(2\pi n)}}}
\tag{30}
$$

5.6. Summary

Five measures that characterize the reverberation effect have been defined in terms of all the parameters of the reverberation unit. Recall that a set of five digital parameters has been previously defined as controlling the whole reverberation unit. The definitions of the five measures are then adapted according to those five parameters.

The measures have been defined using the six delay and six gain parameters of the comb filters. In practice, the relationships between the delays and the gains of the comb filters defined in section 4.2 reduce these numbers to the two parameters of the first comb filter, d_1 and g_1. Likewise, the original fixed parameters of the all-pass filter, d_a and g_a, have been used to define the measures. In practice, the measures are computed for the left channel, where the delay parameters of the all-pass filter are $d_7 = d_a + \frac{m}{2}$ and $g_7 = g_a$. The delay difference parameter m between the all-pass filters at each channel is estimated from d_7. The parameters of the all-pass filter for the right channel are then deduced as follows $d_8 = d_a - \frac{m}{2}$ and $g_8 = g_a$. Note that the cut-off frequency f_c and the wet/dry gain G are the same for each channel. Using Eq. 13, the cut-off frequency can be estimated from the gain of the low-pass filter g_c and the frequency sampling f_s as follows $f_c = \frac{f_s}{2\pi}\arccos\left(2 - \frac{1}{2}(g_c + \frac{1}{g_c})\right)$.

There are in summary five measures-functions of five parameters-variables:

1. $T_{60}(d_1, g_1, f_c, G)$
2. $D(d_1, m)$
3. $C(g_1, f_c, G)$
4. $T_C(d_1, g_1, m, f_c)$
5. $S_C(f_c)$

State-of-art mentions other common measures used to characterize reverberation, such as the *Energy Decay Time*, defined as the total signal energy remaining in the reverberator impulse response p at time t, or the *Inter-Aural Cross Correlation coefficient*, which measures the difference in the sounds arriving at the two channels. However, the Energy Decay Time would have been a redundant measure since it can be used in the definition of some of the measures defined above, and the Inter-Aural Cross Correlation coefficient would have been an inefficient measure when computed from the impulse response. Besides, since five parameters have been defined, a same number of measures, functions of those parameters, seems more suitable for the computations.

Now all the perceptual measures of the reverberation effect are expressed as functions of the digital parameters of the reverberator, the measures need to be mapped back to the parameters so that the reverberation unit can be directly controlled through the measures of the reverberation. Since the functions are not all invertible, especially in d_1 and g_1, and the map parameters-measures is not really bijective, when needed, the parameters are estimated back from the measures by simply using tables of possible values for the parameters d_1 and g_1. Experiments show that the mapping original parameters to measures and then measures to estimated parameters gives really close values of parameters.

Finally, the reverberator can be controlled through the measures of the reverberation. In the process, the five reverberation measures are mapped back to the five independent parameters using the mapping described above. Then those five parameters are used to identify all the parameters using the relations defined in section 4.2. And those parameters are finally used by the reverberation control to generate the desired reverberation effect. Thus the reverberator can generate a reverberation effect based on desired characteristics of the reverberation itself.

6. CONCLUSION

A simple but efficient stereo reverberation control has been developed to simulate reverberation. Inspired from previous classic studies, the reverberator is built using simple digital filters commonly used for simulating reverberation. A total of 21 digital parameters define the whole reverberation control. Five measures commonly used to characterize reverberation have been defined from the impulse response, and formulae are derived to estimate values of these reverberation measures in terms of the digital parameters for the reverberator.

Dependence relations between the digital parameters have led to the need of only five independent parameters to control the whole reverberator, so that the measures have been redefined as five functions of five independent parameters. This mapping parameters-measures is used to let the reverberator to be controlled through the measures of the reverberation, in other words the reverberator can generate a reverberation effect based on desired characteristics of the reverberation itself.

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